

Exercise 10: Plight of the Polar Bear

The polar bear (*Ursus maritimus*) occurs throughout the circumpolar region on the arctic, with their southern range being determined by the amount of sea ice. They rely heavily on sea ice for almost everything: for feeding, breeding, movement and raising young. The long-term persistence of polar bears is threatened by climate change because warming of the arctic region has meant a drastic reduction in sea ice. Reduction in sea ice has adversely affected both survival and reproduction, threatening survival of these beautiful creatures. Under the “business as usual” scenario (i.e., if we do nothing to reduce carbon emission), climate change models predict significant warming of the arctic, and drastic reduction in sea ice. A big question then is: what does future hold for polar bears? In an attempt to address this question, Hunter et al. (2010) applied multistate capture-mark-recapture (CMR) models to data collected during 2001 – 2006, and estimated demographic rates for the female segment of a polar bear population in the southern Beaufort Sea (off the coast of Alaska). They used these parameters to construct and analyze stage-structured, deterministic and stochastic matrix population models.



Population projection matrix for each year of the study are given in the associated R file (matrices, A200X, where X is a year of study).

Basic exploration.... Perform basic matrix model calculations and provide the following information (and answer the questions) (Part I of the code):

1. What are the annual population growth rates? When did the polar bear population grow at the highest and the lowest rates? Plot the population growth rates for each year of the study.

```
> year
[1] 2001 2002 2003 2004 2005
> growth.rates #Table 1, Hunter et al
[1] 1.0590883 1.0614439 1.0362673 0.7649747 0.7991206
```



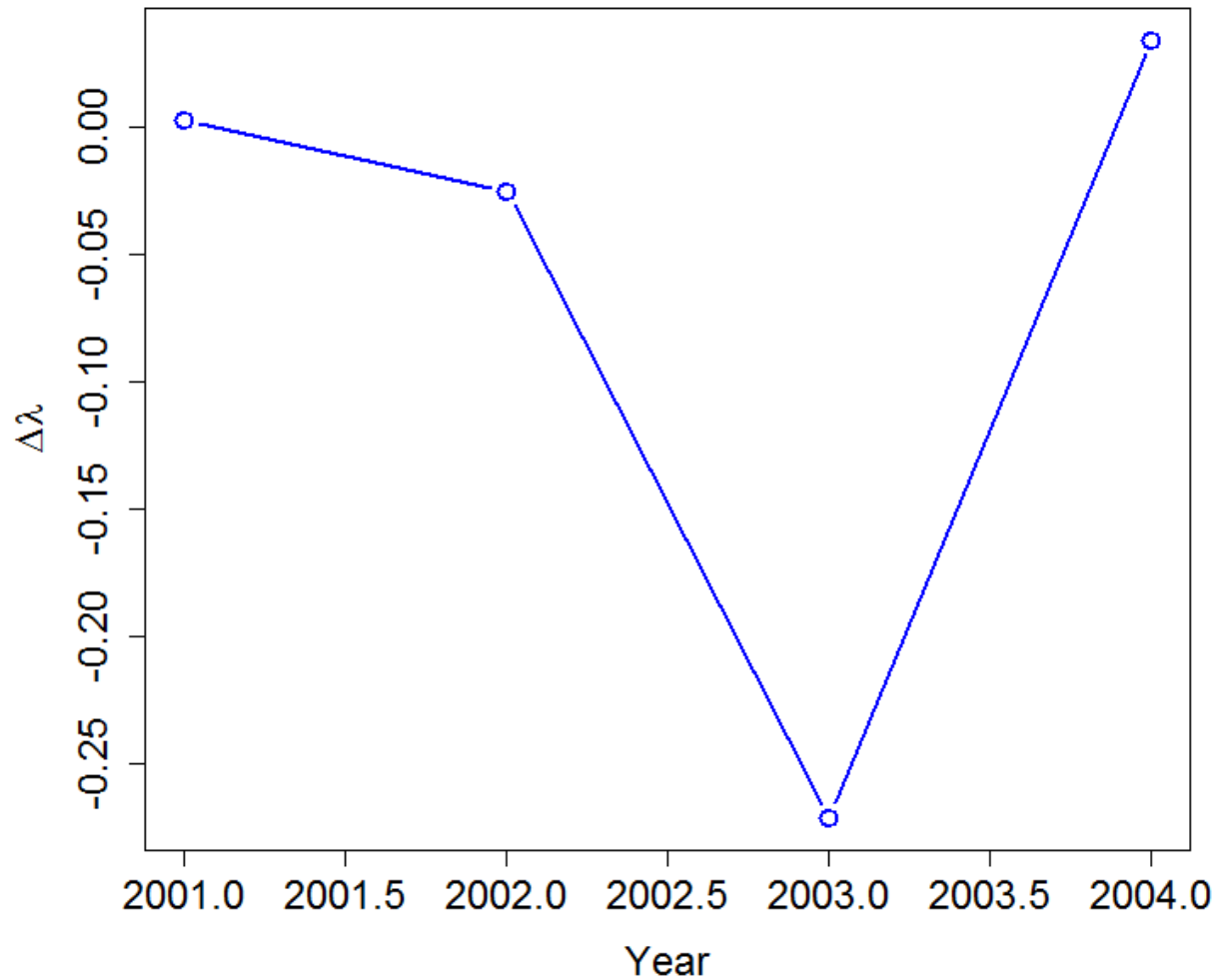
2. Calculate year-to-year differences in population growth rates, and plot the results. When did the largest change in λ occur?

```
3. > c('2001-2002', '2002-2003', '2003-2004', '2004-2005')
```

```

4. [1] "2001-2002" "2002-2003" "2003-2004" "2004-2005"
5. > growth_diff
6. [1] 0.002355608 -0.025176604 -0.271292607 0.034145924

```



7. Calculate the mean, standard deviation, and range of λ across years. What conclusions can you draw based on these results?

```

> mean(growth.rates)
[1] 0.9441789
> sd(growth.rates)
[1] 0.1488217
> min(growth.rates)
[1] 0.7649747
> max(growth.rates)
[1] 1.061444

```

Stochastic demography: estimating long-term growth rate of the polar bear population in an stochastic environment (Part Ib of the code)

8. Calculate stochastic growth rate (Tulja's approximation, as well as simulations + CI) assuming that (use $\text{maxt} \geq 10000$):
 - a. Each year (i. e., matrix) occurs with equal probability
 - b. The first year of the study occurs more frequently than other years
 - c. The last year occurs more frequently than the other years

```
A: > prob
[1] 0.2 0.2 0.2 0.2 0.2
> stoch.results$approx #Tulja's approximation log Lambda s
[1] -0.07419928
> stoch.results$sim #...based on simulation
[1] -0.07201247
> stoch.results$sim.CI #...CIbased on simulation
[1] -0.07479666 -0.06922828
```

```
B: [1] 0.1 0.2 0.2 0.2 0.3
> stoch.results = stoch.growth.rate(matrices = polar.bear.matrices, prob = pr
ob, maxt = 10000)
[1] Calculating stochastic growth at time 1
[1] Calculating stochastic growth at time 10000
> stoch.results$approx #Tulja's approximation log Lambda s
[1] -0.10307
> stoch.results$sim #...based on simulation
[1] -0.0985407
> stoch.results$sim.CI #...CIbased on simulation
[1] -0.10127986 -0.09580154
```

```
C: > prob
[1] 0.3 0.2 0.2 0.2 0.1
> stoch.results$approx #Tulja's approximation log Lambda s
[1] -0.04485334
> stoch.results$sim #...based on simulation
[1] -0.04175844
> stoch.results$sim.CI #...CIbased on simulation
[1] -0.04445391 -0.03906296
```

What conclusions can you draw from these results?

Understanding year-to-year changes in λ : Fixed effect LTRE analysis (Part II of the code)

9. Perform 1-way LTRE analysis, comparing population projection matrix for each year with that in the previous year. Examine the relevant results. How good was your LTRE model? How do you know?

```
> year[2:len]
[1] 2002 2003 2004 2005
> observed_lambda
[1] 1.0590883 1.0614439 1.0362673 0.7649747 0.7991206
> lambda_pred
```

```
[1] NA 1.0614420 1.0362083 0.7624817 0.7993991
```

Excellent fit of the model, because observed and LTRE-predicted values of λ were almost identical!

10. The biggest change in λ occurred during 2003 - 2004, a drop of 0.27! So, let's just focus on understanding what caused that decline.

- Which entry of the projection matrix made the largest contribution to this decline? How do you know?
- Was this decline because of big change in the value or due perhaps to some other reasons? You will have to examine matrix of differences, contribution matrix and perhaps the sensitivity matrix (of the mean matrix) to answer this question.

Relevant results:

Matrix of entry-wise differences:

```
> diff_matrix[[3]]
      [,1] [,2] [,3] [,4] [,5] [,6]
[1,] 0.0000 0.0000 0.0000 0.0000 0.0000 -0.2606
[2,] -0.2837 0.0000 0.0000 0.0000 0.0000 0.0000
[3,] 0.0000 -0.2837 0.0000 0.0000 0.0000 0.0000
[4,] 0.0000 0.0000 -0.2837 0.2527 0.1162 -0.2075
[5,] 0.0000 0.0000 0.0000 -0.4602 -0.1057 0.0000
[6,] 0.0000 0.0000 0.0000 0.0000 -0.2471 0.0000
```

Sensitivity matrix evaluated at the mean values:

```
> sen_mean
      [,1] [,2] [,3] [,4] [,5] [,6]
[1,] 0.01350670 0.01173840 0.01020161 0.4875001 0.09389176 0.03562633
[2,] 0.01554137 0.01350670 0.01173840 0.5609381 0.10803579 0.04099315
[3,] 0.01788255 0.01554137 0.01350670 0.6454389 0.12431051 0.04716843
[4,] 0.02057642 0.01788255 0.01554137 0.7426690 0.14303689 0.05427398
[5,] 0.02150465 0.01868926 0.01624247 0.7761719 0.14948950 0.05672236
[6,] 0.02552297 0.02218151 0.01927751 0.9212061 0.17742285 0.06732140
```

Contribution matrix:

```
> cont_matrix[[3]]
      [,1] [,2] [,3] [,4] [,5] [,6]
[1,] 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 -0.02104092
[2,] -0.01167572 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
[3,] 0.00000000 -0.01167572 0.00000000 0.00000000 0.00000000 0.00000000
[4,] 0.00000000 0.00000000 -0.01167572 0.1175190 0.03014253 -0.02357405
[5,] 0.00000000 0.00000000 0.00000000 -0.2299327 -0.02945778 0.00000000
```

```
[6,] 0.00000000 0.00000000 0.00000000 0.00000000 -0.08241446 0.00000000
```

```
> growth.rates[4] - growth.rates[3]
[1] -0.2712926
> #Sum of LTRE contribution for the same 2 yrs
> sum(cont_matrix[[3]])
[1] -0.2737856
```

LTRE model almost perfectly predicted the decline in λ from 2003 to 2004.

As a reminder, here are the relevant formulas for fixed-effect 1-way LTRE analysis:

$$\lambda^{(m)} \approx \lambda^{(r)} + \sum_{i,j} (a_{ij}^{(m)} - a_{ij}^{(r)}) \frac{\partial \lambda}{\partial a_{ij}} \bigg|_{(\mathbf{A}^{(m)} + \mathbf{A}^{(r)})/2}$$

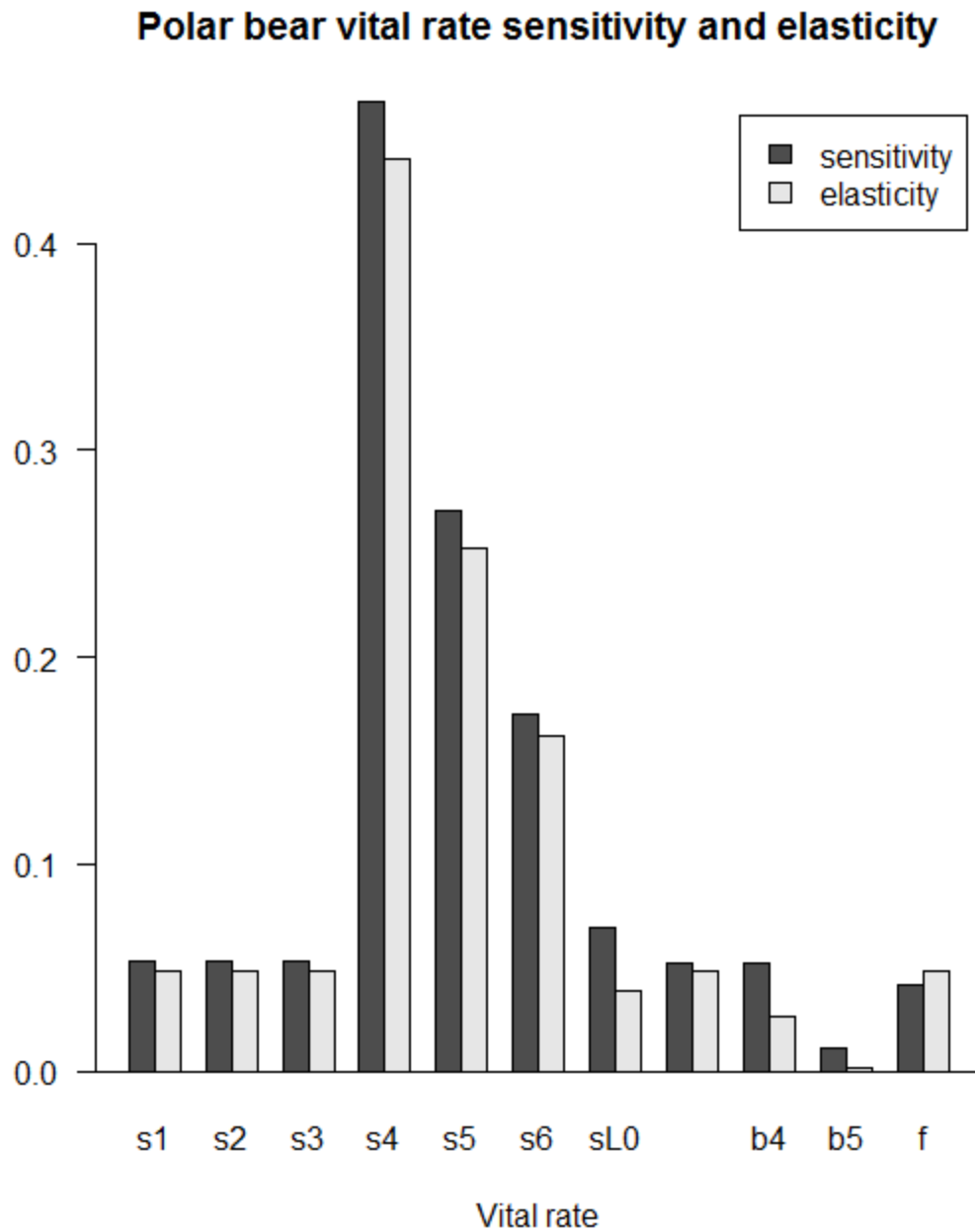
Let $a_{ij}^{(m)} - a_{ij}^{(r)} = \Delta a_{ij}$ and $\lambda^{(m)} - \lambda^{(r)} = \Delta \lambda$.

$$\Delta \lambda \approx \sum_{i,j} \Delta a_{ij} \frac{\partial \lambda}{\partial a_{ij}} \bigg|_{(\mathbf{A}^{(m)} + \mathbf{A}^{(r)})/2}$$

Understanding how year-to-year changes vital rates contribute to $\Delta\lambda$: Fixed effect LTRE analysis with lower-level parameters (Part III of the code)

11. Calculate and plot sensitivity and elasticity of λ to changes in lower-level vital rates for the year 2001. Which vital rate λ is most sensitive to on absolute scale (sensitivity) and proportional or log scale (elasticity)?

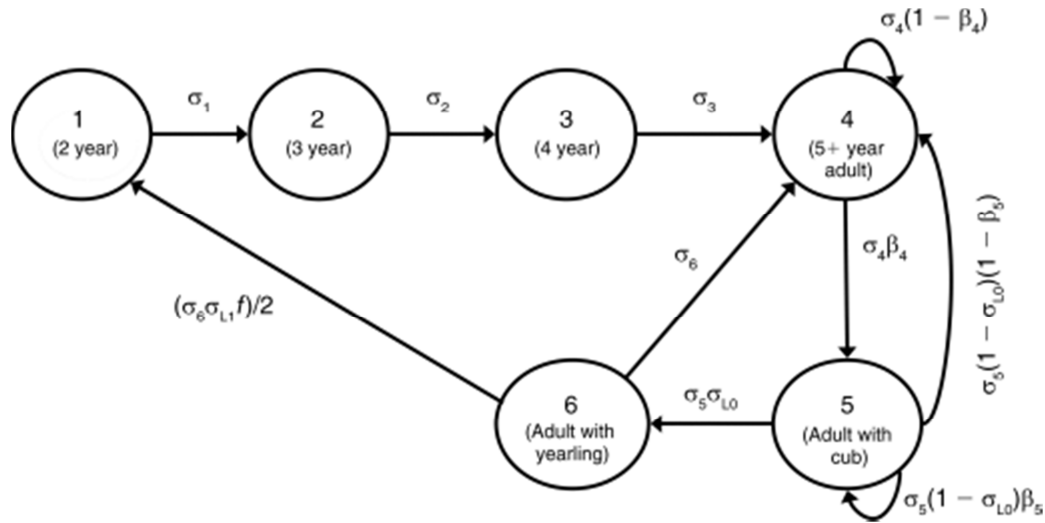
Annual survival rates of stages 4-6 (see life-cycle graph below)



12.

```
> results #Compare these results with Table 2 of Hunter et al. (for 2004)
      estimate sensitivity elasticity vr.diff theta_cont theta_cont.scaled
s1  0.8780000  0.03262254 0.0299780588 0.1080000 0.003523234 0.027233496
s2  0.8780000  0.03262254 0.0299780588 0.1080000 0.003523234 0.027233496
s3  0.8780000  0.03262254 0.0299780588 0.1080000 0.003523234 0.027233496
s4  0.9250000  0.62076586 0.6009811207 0.0740000 0.045936674 0.355076118
s5  0.9120000  0.21223134 0.2025795641 0.0840000 0.017827433 0.137800479
s6  0.9250000  0.11001137 0.1065051389 0.0740000 0.008140841 0.062926155
sL0 0.4535000  0.05642130 0.0267800669 0.2430000 0.013710376 0.105976918
sL1 0.8954341  0.03198737 0.0299780588 0.1025524 0.003280380 0.025356315
b4  0.3010000  0.06080028 0.0191541716 0.4240000 0.025779320 0.199266082
b5  0.0785000  0.00995421 0.0008178388 0.1270000 0.001264185 0.009771752
f   1.1275281  0.02540299 0.0299780588 0.1126809 0.002862431 0.022125695
```

The life-cycle graph to aid in interpretation of the above results:



Polar bear life cycle graph; σ_i is the probability an individual in stage i survives from time t to $t + 1$, σ_{L0} and σ_{L1} are the probabilities that at least one member of a cub-of-the-year or yearling litter, respectively, survives from time t to $t + 1$, f is the expected size of a yearling litter that survives to 2 years, and β_i is the conditional probability, given survival, of an individual in stage i breeding and thereby producing a cub-of-the-year litter with at least one member surviving until the following spring.

References

- Caswell, H. 2001. Matrix population models. Sinauer.
- Cooch, E., R. F. Rockwell, and S. Brault. 2001. Retrospective analysis of demographic responses to environmental change: a lesser snow geese example. *Ecological Monographs* 71:377-400.
- Hunter, C. M., H. Caswell, M. C. Runge, E. V. Regher, S. C. Amstrup, and I. Stirling. 2010. Climate change threatens polar bear populations: a stochastic demographic analysis. *Ecology* 91:2883-2897.